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1974 J. Phys. A: Math. Nucl. Gen. 7 L48

(http://iopscience.iop.org/0301-0015/7/4/005)

J. Phys. A: Math., Nucl. Gen., Vol. 7, No. 4, 1974. Printed in Great Britain. © 1974

LETTER TO THE EDITOR

Diagram techniques in quantum optics: radiative decay and broadening

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Received 15 December 1973

Abstract. We give what is apparently the first rigorous justification of Feynman diagram methods in quantum optics. As an example, we generalize single-atom radiative decay theory to all orders of perturbation of all interactions, by analogy with earlier work on phonon broadening.

Several authors have used Feynman-type diagrams in quantum optics (Low 1952, Fano 1961, Ward 1965, Walls 1971, Knight and Allen 1972, 1973, Chang and Stehle 1971a, b, Duncan 1972, Morawitz 1973, Nishikawa and Aono 1973, Gontier and Trahin 1973). In such treatments the atomic states are represented by boson, fermion or spin operators. However, in each of these cases the diagram formulation requires more justification, since the representation of atomic levels by boson or fermion operators introduces unphysical states into the theory, and since obtaining a Wick theorem for spin operators is not a trivial problem.

We shall represent an atomic level $|n\rangle$ by $a_n^+|0\rangle$ where $|0\rangle$ is a vacuum and a_n^+ obeys fermion (anti)-commutation relations. Both the vacuum and the 'multi-particle' states (eg $a_m^+a_n^+|0\rangle$, $m \neq n$) are unphysical, and therefore give spurious contributions to quantities evaluated using the fermion diagram expansion. Two corrections are needed. First, the use of Abrikosov's procedure (Abrikosov 1965) (adding λ to the energy of each fermion state, and then taking a certain limit in which λ tends to infinity) kills the effect of the multi-particle states. This has the elegant effect of cancelling out diagrams of an easily recognizable class. Second, the vacuum contributions affect the denominator in the expression for any observable, and in the same limit give rise to the breakdown of the linked cluster theorem. Appropriate correction factors should be introduced (Larsen 1972, Verwoerd 1973, Oppermann 1973 and references therein). The first, but not the second, of these corrections has usually been applied by workers unaware of these subtleties. Some aspects of these considerations are depicted in figure 1.

By way of illustration, we now consider the shift and damping of an atomic transition interacting with a reservoir. In this problem one studies the decay of the operator $M_{ij} = a_i^+a_j$ (Lax 1966), or equivalently the quasi-particle resonance of the Green function

$$G_{ij}(E) = \mathscr{F}\langle T_\tau(M_{ij}(\tau)M_{ji})\rangle$$

where T_τ is the chronological operator and $\mathscr{F}$ represents a Fourier transform. Since the same Green function is examined, and since all interactions have their exact analogues, the theory becomes isomorphous to that previously examined in the phonon broadening problem (Stedman 1972). The conclusions in that work may therefore be translated directly to the radiative broadening case. (This represents final confirmation that there is no substance in Brout's distinction between radiative and

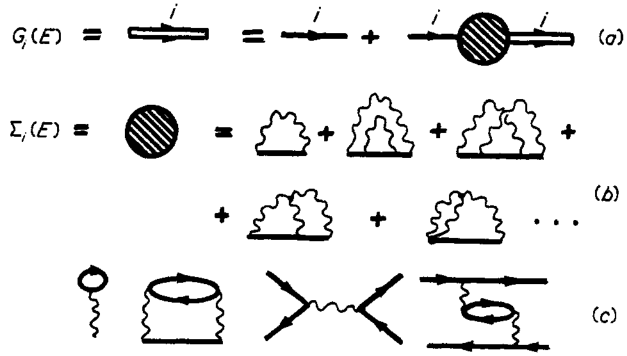

Figure 1. (*a*) Dyson equation for the fermion single-particle Green function $G_i(E)$. A plain line signifies the fermion propagator for zero interaction, and a wiggly line the unperturbed photon propagator. (*b*) The self-energy operator $\Sigma_i(E)$ in the Dyson equation. Knight and Allen (1972), for example, essentially calculate the first term in this perturbation expansion. (*c*) Examples of diagrams cancelled by the Abrikosov procedure.

non-radiative broadening (Brout 1957); cf Stedman 1970.) These conclusions are:

(i) The Weisskopf–Wigner formula for the radiative broadening Γ_{ij} of the $i \to j$ transition:

$$\Gamma_{ij} = W_i + W_j \tag{1}$$

and the Ritz combination principle for the corresponding shift:

$$\Lambda_{ij} = S_i - S_j \tag{2}$$

imply the neglect of electron–hole scattering diagrams (vertex corrections) in the two-particle Green function $G_{ij}(E)$ (figure 2). Here the one-particle shift and width (S_i, W_i)

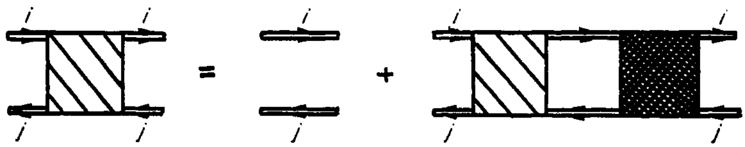

Figure 2. The Bethe–Saltpeter type equation for the fermion two-particle Green function. The cross-hatched box represents an irreducible scattering vertex. The first term on the right-hand side represents the approximation in which equation (1) holds, ie that of ignoring the scattering vertex.

are to be defined as the real and imaginary parts of a lorentzian approximation† to the self-energy Σ_i of the one-particle Green function $G_i(E) = \mathscr{F}\langle T_\tau(a_i(\tau)a_i^+)\rangle$.

†In Stedman (1972) the lorentzian approximation was chosen so that in studying a certain possible decay mode, only the leading term in the coefficient is considered. Thus we consider only second-order perturbation contributions to direct (one-photon linear) atomic absorption rates, etc. This permits an otherwise general proof of (ii) and (iii), and it also eliminates the effect of unlinked diagrams.

(ii) The effect of the scattering vertex diagrams is to modify equation (1), but not equation (2) which is always valid.

(iii) The contributions of the scattering vertex diagrams to equation (1) may be interpreted physically as the effects of elastic interaction with photons, eg (but not only) elastic Raman scattering.

These results generalize those of Lax (1966), who showed that, to second order,

$$\Gamma_{ij} = \tfrac{1}{2}(\Gamma_i + \Gamma_j) + \Gamma_{ij}{}^{\text{ph}} \tag{3}$$

where $\Gamma_{ij}{}^{\text{ph}}$ reflected the effects of elastic (phase) interaction†. Lax also discussed 'anomalous shifts' which would disobey the Ritz combination principle, and was able to prove that second-order anomalous shifts vanished only when the system obeyed time-reversal symmetry and when there was no interaction among the reservoir modes. We emphasize that our proof is valid, independently of time-reversal properties, for all types of atom–reservoir interactions and internal interactions, including anti-resonant terms and non-linear effects of all kinds, and is valid for all orders of perturbation. Thus our generalization of the Weisskopf–Wigner theory (equations (2) and (3)) is not confined to RWA (cf Knight and Allen 1973). We note that the leading term in S_i in the phonon case (Stedman 1972) is just that found by Knight (1972) in the photon case, with each reservoir in thermal equilibrium (ie T^4) (cf also Gontier and Trahin 1973).

Equation (3) has also been derived in the context of pressure broadening (Fano 1963); in this case elastic electron scattering gives rise to a non-separable width contribution.

Already there is experimental evidence that equation (1) is inadequate for the phonon case (Stedman and Cade 1973). In the photon case, one is not dependent solely on thermal radiation, and it has proved possible to stimulate one atomic transition while studying the lineshape from another (Bose and White 1971 and references therein). It is therefore unsatisfactory to assume the accuracy of equation (1) in analysing such experiments (as does Holt 1968). We would rather suggest that such experiments are ideally suited to test our predictions. Elastic scattering effects should be relatively important if the stimulating laser frequency is slightly different from that of the atomic transition, so that the direct absorption effects are eliminated.

I am grateful to Drs D F Walls, C W Gardiner, F Ansbacher and D Smith for discussions.

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†Note that our W_i is not half Lax's Γ_i, since W_i includes some of the effects of elastic scattering. Note also that we do not follow Lax in identifying only $\Gamma_{ij}{}^{\text{ph}}$ with no-phonon line broadening. All the terms in Γ_{ij} have analogues in the no-phonon line broadening case.

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